Integrali indefiniti (con i differenziali)

Integrali immediati e notevoli: ponendo $\boxed{\otimes \ = \ g(x)}$,

1. $\displaystyle\int k \, d\otimes = \ k \otimes + c$

2. $\displaystyle\int \otimes^{\alpha} \, d\otimes = \frac{\otimes^{\alpha+1}}{\alpha+1} + c \qquad (con \ \ \alpha \neq -1)$

3. $\displaystyle\int \frac{1}{\otimes} \, d\otimes = \ln|\otimes| + c$

4. $\displaystyle\int \sin\otimes \, d\otimes = -\cos\otimes + c$

5. $\displaystyle\int \sin^2\otimes \, d\otimes = \frac{1}{2}\left(\otimes - \sin\otimes\cos\otimes\right) + c$

6. $\displaystyle\int \frac{1}{\sin\otimes} \, d\otimes = \ln\left|\tan\frac{\otimes}{2}\right| + c$

7. $\displaystyle\int \frac{1}{\sin^2\otimes} \, d\otimes = -\cot\otimes + c$

8. $\displaystyle\int \cos\otimes \, d\otimes = \sin\otimes + c$

9. $\displaystyle\int \cos^2\otimes \, d\otimes = \frac{1}{2}\left(\otimes + \sin\otimes\cos\otimes\right) + c$

10. $\displaystyle\int \frac{1}{\cos\otimes} \, d\otimes = \ln\left|\tan\left(\frac{\otimes}{2} + \frac{\pi}{4}\right)\right| + c$

11. $\displaystyle\int \frac{1}{\cos^2\otimes} \, d\otimes = \tan\otimes + c$

12. $\displaystyle\int e^{\otimes} \, d\otimes = e^{\otimes} + c$

13. $\displaystyle\int a^{\otimes} \, d\otimes = \frac{a^{\otimes}}{\ln a} + c$

14. $\displaystyle\int \frac{1}{1+\otimes^2} \, d\otimes = \operatorname{atan}\otimes + c$

15. $\displaystyle\int \frac{1}{1-\otimes^2} \, d\otimes = \frac{1}{2}\,\ln\left|\frac{1+\otimes}{1-\otimes}\right| + c$

16. $\displaystyle\int \frac{1}{\alpha^2+\otimes^2} \, d\otimes = \frac{1}{\alpha}\operatorname{atan}\frac{\otimes}{\alpha} + c$

17. $\displaystyle\int \frac{1}{\alpha^2+(\otimes+\beta)^2} \, d\otimes = \frac{1}{\alpha}\operatorname{atan}\frac{\otimes+\beta}{\alpha} + c$

18. $\displaystyle\int \frac{1}{\sqrt{1-\otimes^2}} \, d\otimes = \operatorname{asin}\otimes + c$

19. $\displaystyle\int \frac{1}{\sqrt{1+\otimes^2}} \, d\otimes = \ln(\otimes + \sqrt{1+\otimes^2}) + c$

20. $\displaystyle\int \frac{1}{\sqrt{\alpha^2-\otimes^2}} \, d\otimes = \operatorname{asin}\frac{\otimes}{|\alpha|} + c$

21. $\displaystyle\int \sqrt{\alpha^2-\otimes^2} \, d\otimes = \frac{1}{2}\left(\alpha^2\operatorname{asin}\frac{\otimes}{|\alpha|} + \otimes\sqrt{\alpha^2-\otimes^2}\right) + c$

Differenziale di $g(x)$**:** $\boxed{g'(x)\,dx = d\,g(x)}$

Esempi

$$\frac{1}{2}\int \otimes^5\,d\otimes = \frac{1}{2}\,\frac{\otimes^6}{6} + c$$
$$\ldots con\ \otimes = 2x-3$$

a) $$\int (2x-3)^5\,dx = \frac{1}{2}\int (2x-3)^5\,\underbrace{2\,dx}_{g'(x)\,dx} = \frac{1}{2}\int (2x-3)^5\,\underbrace{d(2x-3)}_{d\,g(x)} = \frac{(2x-3)^6}{12} + c$$

$$-\frac{1}{3}\int e^{\otimes}\,d\otimes = -\frac{1}{3}\,e^{\otimes} + c$$
$$\ldots con\ \otimes = \cos 3x$$

b) $$\int e^{\cos 3x}\sin 3x\,dx = -\frac{1}{3}\int e^{\cos 3x}\,\underbrace{(-3)\sin 3x\,dx}_{g'(x)\,dx} = -\frac{1}{3}\int e^{\cos 3x}\,\underbrace{d\cos 3x}_{d\,g(x)} = -\frac{1}{3}\,e^{\cos 3x} + c$$

$$2\int \otimes^{-2}\,d\otimes = -\frac{2}{\otimes} + c$$
$$\ldots con\ \otimes = \ln x$$

c) $$\int \frac{2}{x\,\ln^2 x}\,dx = 2\int (\ln x)^{-2}\,\underbrace{\frac{1}{x}\,dx}_{g'(x)\,dx} = 2\int (\ln x)^{-2}\,\underbrace{d\ln x}_{d\,g(x)} = -\frac{2}{\ln x} + c$$

$$\frac{1}{2}\int 3^{\otimes}\,d\otimes = \frac{3^{\otimes}}{2\ln 3} + c$$
$$\ldots con\ \otimes = \tan 2x$$

d) $$\int \frac{3^{\tan 2x}}{\cos^2 2x}\,dx = \frac{1}{2}\int 3^{\tan 2x}\,\underbrace{\frac{2}{\cos^2 2x}\,dx}_{g'(x)\,dx} = \frac{1}{2}\int 3^{\tan 2x}\,\underbrace{d\tan 2x}_{d\,g(x)} = \frac{3^{\tan 2x}}{2\ln 3} + c$$

e) $$\int \frac{1}{5+e^x}\,dx = \int \frac{1}{e^x(5e^{-x}+1)}\,dx = \int \frac{e^{-x}}{5e^{-x}+1}\,dx =$$

$$-\frac{1}{5}\int \frac{1}{\otimes}\,d\otimes = -\frac{1}{5}\ln|\otimes| + c$$

$\ldots con\ \otimes = 5e^{-x}+1$

$$-\frac{1}{5}\int \frac{1}{5e^{-x}+1}\underbrace{(-5)e^{-x}\,dx}_{g'(x)\,dx} = \quad -\frac{1}{5}\int \frac{1}{5e^{-x}+1}\underbrace{d(5e^{-x}+1)}_{d\,g(x)} \quad = -\frac{1}{5}\ln(5e^{-x}+1) + c$$

$$\frac{2\sqrt{3}}{3}\int e^{\otimes}\,d\otimes = \frac{2\sqrt{3}}{3}\,e^{\otimes} + c$$

$\ldots con\ \otimes = 2+\sqrt{3x}$

f) $$\int \frac{e^{2+\sqrt{3x}}}{\sqrt{x}}\,dx = \frac{2\sqrt{3}}{3}\int e^{2+\sqrt{3x}}\underbrace{\frac{3}{2\sqrt{3}\sqrt{x}}\,dx}_{g'(x)\,dx} = \quad \frac{2\sqrt{3}}{3}\int e^{2+\sqrt{3x}}\,\underbrace{d(2+\sqrt{3x})}_{d\,g(x)} \quad = \frac{2\sqrt{3}}{3}\,e^{2+\sqrt{3x}} + c$$

$$\frac{3}{4}\int \frac{1}{\frac{3}{2}+\otimes^2}\,d\otimes = \frac{3}{4}\sqrt{\frac{2}{3}}\,\text{atan}\left(\sqrt{\frac{2}{3}}\otimes\right) + c$$

$\ldots con\ \otimes = x^2$

g) $$\int \frac{3x}{3+2x^4}\,dx = \frac{3}{2}\int \frac{x}{\frac{3}{2}+x^4}\,dx = \frac{3}{4}\int \frac{1}{\frac{3}{2}+x^4}\,\underbrace{2x\,dx}_{g'(x)\,dx} = \qquad \frac{3}{4}\int \frac{1}{\frac{3}{2}+x^4}\,\underbrace{dx^2}_{d\,g(x)}$$

$$= \frac{\sqrt{6}}{4}\,\text{atan}\left(\sqrt{\frac{2}{3}}x^2\right) + c$$